Rewrite: December 11, 2006
February 16, 2006

Special and General Relativity Analogies

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Contents

Introduction

Einstein founded the theory of special relativity on two postulates: as with regard to the principle of covariance and constant light speed. By covariance, any one of all observers moving relative to each other can be considered as relatively at rest. The effects of relative motion are therefore the same for all observers. Clocks, for instance, are relatively slow due to them being in relative motion. Since each observer perceives the other observer's clock as slow, there is no unique distinction to be made between them.

The principle of covariance also includes constant light speed. Light speed is the same relative to all observers regardless of any unknown velocity of the system in which they move. Observers thus determine light speed the same relative to the system in which they move even if one moves nearly at light speed relative to the other. Apart from the principle of covariance, however, light speed is unique in its own right. Regardless of its energy, all light in a vacuum of gravitational free space moves at the same rate whereas material objects move at various speeds relative to each other. In effect, relative spacetime conditions conform to the covariance of constant light speed such that there is no distinction made of any system apart from another.

This constant nature of light speed has been verified many times over by countless experiments. However, there is questionable issue with regard to interpreting theory in consistent manner. Gravity affects the spacetime path of light. As a consequence, special relativity is relegated to being valid only as a special case, as where the influence of gravity is negligible for observable effect. This result further implies the non-application of constant light speed in a gravitational field. It implies the relative spacetime of gravity differs from the relative spacetime of relative motion in way of light speed varying according to the relative strength of the gravitational field in which it moves.

Neither Einstein nor anyone else has hitherto deduced the relative spacetime of general relativity to the relative spacetime of special relativity. This condition of relegating special relativity to a limited validity is nonetheless challenged, as it is hereby claimed the relative spacetimes of special and general relativity are essentially of the same form. All relative effects due to the presence of a gravitational field (except for one possible exception) are thus analogous to the relative effects due to relative motion: whereby, the speed of light is constant whether it moves in a gravitational field or in gravitational free space.

Special and general relativity are of the same form if all conditions of a gravitational field are analogous to the conditions of relative motion. A clock in a gravitational field is thus slow by reason a clock in relative motion is slow. The analogical condition also requires light speed is a constant in a gravitational field as well as in gravitational free space. This condition is not a formidable obstacle inasmuch as speed is somewhat arbitrarily defined, as according to the simple equation $v = s/t$. Instead of light speed, $v = c$, varying in the field, the relative spacetime parameters s and t vary instead. However, a more formidable obstacle is in relation to special relativity itself.

The more formidable obstacle is with regard to the established relativistic definition of kinetic energy in relation to the escape velocity of the gravitational field. This definition is here replaced with a modified Newtonian escape velocity conforming to relative spacetime conditions.

There is another discrepancy. Being the two theories are of the same form, light speed further represents a limiting aspect for the gravitational potential as well as for relative motion. However, the final result is surprisingly different. What this limiting aspect of gravity actually entails differs somewhat in nature from that of relative motion. It differs insofar as two opposing limits converge at the limit of the gravitational potential in relation to one-half light speed instead of light speed. This non-analogical condition of gravity occurs because of the nature of gravity or spacetime curvature canceling itself out, as the outer edge of the field become tantamount to the center of the field. The result uniquely reaffirms Einstein's cosmological model of the universe curving in on itself whereby the observer is always at its center.

This non-analogy condition also requires further interpretation of the Schwarzschild radius from which black holes and singularities have been derived. The black hole is still a reality but the so-called singularity, on which some expansion theories of the universe are based, does not occur.

Constant Light Speed

Physicists of the 19th century considered light as waves propagating through a medium called the ether. They believed all light waves of any frequency and length progress forward through the ether at the same speed. They further believed other velocities relative to the ether could be ascertained in relation to light speed. However, the result of experiment indicates light speed is the same regardless of any unknown velocity of the object or observer relative to the ether.

How does light move at the same speed relative to all observers? A few physicists, such as Lorentz, explained it as matter being affected in a unique way by its motion through the ether. Matter is contracted in the direction of relative motion and inertly slower to respond. Einstein did not accept this explanation because he regarded the ether as superfluous for the mathematical formulation of scientific theory, but he still formulated the principle of special relativity in a way the relative effects of matter in relative motion are retained. Therefore, matter is contracted in the direction of relative motion, clocks in relative motion are relatively retarded, and the inertia of matter in relative motion is relatively increased. As a consequence, the relativity of spacetime conforms to the nature of constant light speed.

A particular consequence of relative spacetime is Einstein's addition of velocities theorem:

$$v_{12} = \frac{v_1 + v_2}{1 + v_1 v_2 c^{-2}}$$

By this theorem, system C moving at velocity v_2 relative to system B moving at velocity v_1 relative to system A is not $v_1 + v_2$; it is slightly less. It is less by the factor $(1 + v_1v_2c^{-2})$. Although contrary to common sense, this mathematical formula still agrees with the concept of relative spacetime in view of constant light speed. Let either v_1 or v_2, for instance, be the same as light speed. Hence

$$v_{12} = \frac{c + v_2}{1 + c v_2 c^{-2}} = \frac{c + v_2}{1 + v_2 c^{-1}} = \frac{c\left(1 + v_2 c^{-1}\right)}{1 + v_2 c^{-1}} = c$$

$$v_{12} = \frac{v_1 + c}{1 + v_1 c c^{-2}} = \frac{v_1 + c}{1 + v_1 c^{-1}} = \frac{c\left(1 + v_1 c^{-1}\right)}{1 + v_1 c^{-1}} = c$$

Light speed added to light speed is also

$$v_{12} = \frac{c + c}{1 + c c c^{-2}} = \frac{2c}{1 + 1} = c$$

Therefore, the speed of light is the same relative to systems A, B, C and so forth, and even the same relative to light speed. It is the same no matter what.

The Clock Paradox

By the clock paradox, a returning clock is found to have been slower than one that stayed home. This result seems contrary to the principle of covariance but is explained by way of acceleration. One clock accelerates to change relative direction and return home whereas the other one does not. The two events are thus asymmetrical such that the principle of covariance applies in a more complex way. If the traveling observer does not reverse direction to return home, for instance, but the system of the stay at home observer accelerates to catch up, then the previous stay at home observer's clock is found to be slow.

Although the clock paradox has been well explained in countless books, there are further implications of it with regard to the measure of light speed, acceleration and gravity. Consider, for instance, the event of a light signal emitted away from two clocks at the time of their departure and reflected back to be received at the time of their reunion. Since the event is relatively simultaneous, two clocks of different rates obviously determine different times for the event. Moreover, let the light move perpendicular to the direction of relative motion between the two clocks. Because distance perpendicular to the direction of relative motion remains relatively the same, as according to theory, it seems the slower clock in relative motion around the other determines light speed as relatively faster.

How is the constant nature of light speed maintained? Unlike inertial systems of relative motion, acceleration is measurable. The returning clock, for instance, accelerated when it changed direction. This change in direction is experienced, in effect, differently than the stay-at-home observer merely witnessing it. This experience is also consistent with a mathematical analysis in the sense the returning clock needs to move faster to catch up even if the stay-at-home clock were initially in relative motion. The clock in acceleration is therefore determined to be slow such that the nature of constant light speed is simply maintained by analytical deduction. In consequence, it is more consistently correct to say the spacetime of the traveling observer, the one who accelerated, relatively expanded such that the distance and duration of events in relative spacetime appears relatively shorter.

Suppose, further, the acceleration of one clock is circular around the other such that their distance of separation remains constant. In this case, a to-there-and-back propagation of light from one clock is timed differently than the same to-there-and-back propagation of light from the other. This condition is similar to the clock paradox in that light moves perpendicular to the direction of relative motion and one clock accelerates whereas the other one does not.

Say these two clocks move relative to another. Even if the accelerating clock is relatively slower in one direction, its average motion due to the greater speed in the opposite direction is greater. By covariance, all systems are the same, such that the circling clock is slow by relative motion no matter at what speed the systems moves as a whole. It is nevertheless more consistent to say the same circular spacetime of the accelerating observer is relatively greater with respect to that observer moving a relatively shorter distance within that relatively greater spacetime, as opposed to claiming light speed is relatively faster.

Light Speed and Gravity

What if the event of acceleration is caused by gravity? By special relativity, all events are related to constant light speed in a mathematically consistent manner, but this consistency is not maintained by certain implications of general relativity. By them, light speed varies according to the relative strength of the gravitational field in which it propagates, as opposed to constant light speed in a relatively expanded spacetime.

This inconsistency of theory seems unnecessary in view of general relativity being founded on a postulate that equally compares events in free fall to those in relative motion. Indeed, by the principle of equivalence, internal events in a relatively homogeneous gravitational field are determined locally according to the conditions of special relativity. Inasmuch as light and the observer fall at the same rate, the constant nature of light speed is locally maintained. It varies in view of a non-local observer not in free fall, but the prevention of free fall is a condition of acceleration analogous to that of changing direction.

In effect, a clock in a gravitational field is relatively slow in comparison to one in gravitational free space. This relativistic effect is analogous to a relatively slow clock in relative motion. Despite this analogy, however, it is still implied light speed varies according to the relative strength of the gravitational field in which it propagates.

Why is general relativity interpreted differently? Einstein merely chose to formulate general relativity according to the principle of equivalence and considered the validity of special relativity as only applying insofar as the inhomogeneous effect of gravity on spacetime is negligible. More generally, tidal effects compromise the condition of not being aware of free fall. The moon's gravitational pull on earth, for instance, varies from place to place and gives rise to ocean tides. This local awareness of internal effect of an inhomogeneous field of gravity is more generally referred to as spacetime curvature. Gravity occurs because material objects and light merely follow the path of spacetime curvature, which is due to the presence of mass distorting all spacetime around it.

Einstein thus chose a simpler way, a geometrical one, of describing relative spacetime curvature whereby he neglected to include constant light speed as conditional to it. Because gravity affects the path of light, gravity determines the relativity of spacetime apart from constant light speed. However, including constant light speed as part of the equation does not change the mathematical formulation of general relativity. It is the same because distance, time and speed are interrelated. Since speed is a measure of distance per time, a relatively greater distance per time is mathematically equivalent to a relatively slower speed. Varying light speed in place of relative time is merely arbitrary if this greater distance is relative, which is to be shown as such with regard to an internal relativistic expansion of the gravitational field.

Including constant light speed as conditional maintains consistency of theory. It is consistent inasmuch as the conditions of spacetime curvature due to the presence of mass and relative spacetime apart from gravity are analogous. According to special relativity, for instance, the speed of matter can neither equal nor exceed that of light in a vacuum. The analogy with spacetime curvature is with regard to light speed representing a limiting condition by which the escape velocity of the gravitational field can neither equal nor exceed. By established interpretation, it represents the condition of a black hole from which nothing can escape. By another interpretation, as here proposed, it represents a possible limit to which the presence of mass can neither equal nor exceed. In other words, spacetime curvature is itself subject to the same conditions to which relative masses in relative motion are. However, the true nature of this limit is still open to interpretation. Another interpretation here proposed is the condition whereby limits converge at one-half light speed instead of at light speed itself.

In a way, the established interpretation is conditional to constant light speed. It is conditional with regard to constant light speed being maintained as an internal condition of free fall by local observers. An escape velocity also pertains to an observer's local position in the field. For instance, if spacetime curvature is such that light cannot find its way out, then everything within that spacetime curvature is confined as well.

The alternative interpretation is proposed as being even more consistent. It is more consistent in the sense the relative spacetime of the gravitational field, or spacetime curvature due to the presence of mass, relatively expands in analogous manner relative spacetime of relative motion does. No black hole need occur if relative spacetime expansion prevents a relative mass-density from occurring in way of equaling or exceeding the limit of the escape velocity at light speed. In conclusion, however, as due to the possibility of the limit only being at one-half light speed instead of light speed, a black hole is permitted in way of it becoming a finite visible universe within itself. Only the so-called singularity is inconsistent with theory.

Comparing Form

The major difference between special relativity and general relativity, as according to the established interpretation of the latter, is with regard to light speed. According to special relativity, light speed in a vacuum of gravitational free space is constant and the same relative to all observers. According to general relativity, it is regarded as varying according to the relative strength of the gravitational field in which it moves. This variance is derived from the Schwarzschild metric, a particular solution to Einstein's general field equations relating Newtonian parameters according to spacetime curvature.

The Schwarzschild metric is of the form

$$ds^2 = \left(1 - \frac{2GM}{r}\right) dt^2 - \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - \left(\sin^2\theta\right) d\varphi^2 .$$

It is valid for an infinitesimal spacetime interval ds where the field is relatively homogeneous. Its form is in contrast to a special relativity metric for standard invariance of the interval ds:

$$ds^2 = dt^2 - dx^2 .$$

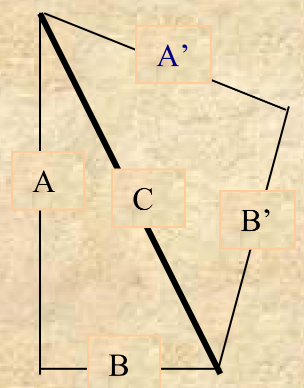
The Schwarzschild metric is not directly deducible to this standard invariance, but, rather, to a similar metric with regard to acceleration of the form

$$ds^2 = dt^2\left(1 - v^2\right) - dx^2\left(1 - v^2\right).$$

This similar metric is merely that with regard to internal spacetime expansion due to the clock of the observer in a constant state of acceleration being relatively slower. The deduction of the Schwarzschild metric is with regard to substituting internal spacetime expansion for variable light speed.

Although these metrics are slightly of different form, the difference is explainable as describing different events. The one for standard invariance describes events according to the viewpoint of an observer relatively at rest in an inertial system whereas the others are according to the viewpoint of an observer in a state of acceleration, either by gravity or otherwise. Although the one for relative motion of acceleration still appears different in form from that of the Schwarzschild metric in its present form, it can still be shown the difference is merely with regard to the interpretation of light speed and relative spacetime, as to whether light speed in a gravitational field is slow or spacetime is relatively expanded instead.

The interpretation is simply according to the nature of the metric. In physics, metric refers to an equation that relates coordinate lengths as infinitesimal increments and allows for the calculation of them in terms of an invariant. An invariant is something staying the same even though its internal components can change. A particular length of a hypotenuse of a right triangle, for instance, is representative of a whole family of right triangles of the same hypotenuse. According to the Pythagorean Theorem, the hypotenuse squared is equal to the sum of the squares of the two other sides. Since the hypotenuse is common for a whole family of right triangles, the length of the hypotenuse squared is equal to the sum of the lengths squared of the other two sides of any of the triangles in the family. The length of the hypotenuse can thus be determined by measuring the lengths of the other two sides of any one of the triangles in the family. The hypotenuse further represents an invariant for all determinations of lengths in the same family of right triangles.



$$C^2 = A^2 + B^2 = A'^2 + B'^2$$

A similar invariance for relative spacetime coordinates in view of constant light speed has been derived from the postulates of special relativity. It is of the form

$$s^2 = t^2 - x^2 = t'^2 - x'^2 \quad \text{or} \quad s^2 = x^2 - t^2 = x'^2 - t'^2$$

It differs from that of the Pythagorean Theorem in that the square of either space or time coordinates are subtracted from the other instead of added. The metric is merely the incremental distance and time of such coordinates.

Invariance of the Schwarzschild metric is also according to an infinitesimal volume of space within a relatively homogeneous gravitational field where dr is an increment of distance, dt is an increment of time, and ds is some incremental equivalent to their invariance. The gravitational strength of Earth, for instance, only slightly changes a foot higher from the ground than it does many miles up. At an inch difference it is even more homogeneously the same and converges to a homogeneous state for an infinitesimal distance.

The Schwarzschild metric also derives from Einstein's field equations as an interpretation of Newton's inverse-square-law for gravity in terms of spacetime curvature. In Newtonian Mechanics, there is a critical velocity needed to escape from the field at any particular position. It is calculated to be the square root of 2 times the circular speed for gravitational free fall. Accordingly, the escape velocity at radius r from the center of mass M is the square root of 2GM/r, where G is the gravitational constant. Conversely, the escape velocity squared is twice the Newtonian potential of GM/r. Therefore, the relativistic factor in the Schwarzschild metric, as with regard to the term 2GM/r, is conditional to the escape velocity rather than directly to the Newtonian potential.

The increments dt and dr represent time and distance, respectively, and the last two terms with Greek letters are trigonometric quantities referring to polar coordinates in place of the standard coordinates y and z in flat spacetime. Flat spacetime means a relatively gravitational-free spacetime that can be calculated according to the conditions of special relativity.

The letter c represents light speed in a vacuum of gravitational free space. However, coordinate lengths of space and time are conveniently chosen so that light speed is defined as unity, c = 1, which does not appear in the equation. The factor (1 – 2GM/r) is, for instance, actually ($1 - 2GM/rc^2$). An explanation of this factor is now given for the purpose of later illustrating the significance of its presence.

The factors ($1 - 2GM/rc^2$) and ($1 - v^2/c^2$) are generally referred to as meaning relativistic. Relative means comparable to, and relativistic is a comparable effect of being in either relative motion or a gravitational field. The relativistic factor in the Schwarzschild metric thus distinguishes the relative difference in something such as mass-energy, distance or time in a gravitational field as compared to being in gravitational free space. In particular, if $2GM = rc^2$, then $2GM/rc^2$ is one and the relativistic factor ($1 - 2GM/rc^2$) is zero. The radius $r = 2GM/c^2$ is known as the Schwarzschild radius of mass M that indicates the boundary of a black hole where nothing can escape.

The same equations used to predict the existence of black holes supposedly further predict the condition of a singularity whereby all matter in the universe can condense to an infinitesimal point. A singularity is indicated at the distance $r = 2GM/c^2$ only because the quantity of time $dt(1 - 2GM/rc^2)$ becomes zero and the quantity $dr/(1 - 2GM/rc^2)$ becomes a distance divided by zero. Since a quantity divided by zero is undefined in mathematics, except for the condition of convergence in calculus, the singularity can be interpreted as meaning nonsense.

Inasmuch as there is nowhere else to go except more inside after entering into the field at the Schwarzschild radius, it has also been interpreted as indicating the inevitable result of all matter within the field being condensed within an infinitesimal volume of space, a point so to speak. In contrast, the black hole exists within the boundary of $r = 2GM/c^2$, which is also referred to as the event horizon.

How a black hole derives from the Schwarzschild metric is explained in view of invariance. In gravitational free space, standard invariance of special relativity is of the form $s^2 = t^2 - x^2$. Let s be zero. Hence

$$0 = t^2 - x^2 \qquad x^2 = t^2 \qquad \frac{x^2}{t^2} = 1$$

$$\frac{x}{t} = 1$$

For an interpretation of this unit velocity, it is light speed, as where x is the unit distance light moves in time t. Hence

$$\frac{x}{t} = c \qquad x = ct \qquad x^2 = c^2 t^2$$

$$0 = c^2 t^2 - x^2$$

This mathematical relation therefore represents the event of light moving the unit distance x within unit time t.

Light speed in a gravitational field is similarly derived according to the Schwarzschild metric. In ignoring polar coordinates, the metric obtains the form

$$ds^2 = \left(1 - \frac{2GM}{r}\right)dt^2 - \frac{dr^2}{1 - \frac{2GM}{r}}$$

According to special relativity, the simultaneity of events, as seen by a particular observer, is determined by light speed. If the particular event in question is a particle of light, then the difference between the displacement of the particle and its observation is zero. Hence, the interval ds is zero and the equation becomes

$$0 = \left(1 - \frac{2GM}{r}\right)dt^2 - \frac{dr^2}{1 - \frac{2GM}{r}}$$

$$\frac{dr^2}{1 - \frac{2GM}{r}} = \left(1 - \frac{2GM}{r}\right)dt^2$$

$$\frac{dr^2}{dt^2} = \left(1 - \frac{2GM}{r}\right)^2$$

$$\frac{dr}{dt} = 1 - \frac{2GM}{r}$$

The relative speed of light in a gravitational field is thus less than unity.

Significantly, in the absence of a gravitational field, 2GM is zero and the relativistic factor numerically reduces to the number one, which, in this case, is the speed of light in gravitational free space as one unit of space divided by one unit of time. At the opposite extreme, however, where 2GM = r, as where the escape velocity becomes unity, then the relativistic factor and light speed becomes zero. Light propagation is thereby confined. Moreover, special relativity also stipulates nothing moves in an observable manner faster than light speed. Since light speed is zero and no information of events moves any faster, all mass-energy is confined as well. The black hole thus appears inevitable considering the potential magnitude of M is unlimited. However, M in a particular volume of space is not unlimited if the volume of space relatively expands instead.

Converting Form

The result above indicates a variable light speed such that general relativity appears to be of different form than the spacetime of special relativity. Special relativity, which is included as a special case of general relativity, is formulated according to the postulate of constant light speed. However, the latter is not formulated according to the same postulates of the former; it is formulated according to the principle of equivalence by which spacetime curvature is primarily subject to the conditions of the gravitational field. The two theories by this interpretation are different in form because relative spacetime is not interpreted according to constant light speed in all respects of a gravitational field. However, to say again, it is not the principle of equivalence at fault; rather, there is an inconsistency with regard to the interpretation of theory, as neglecting to include constant light speed as part of it.

It is not impossible to formulate general relativity according to the form of special relativity. All that is needed is for spacetime curvature to be described according to constant light speed. This requirement is achieved by merely interpreting spacetime coordinates as merely expanded instead of as light speed being slower in a gravitational field. Thus, in special relativity, spacetime is distorted by relative motion and this distortion is according to constant light speed; in general relativity, spacetime curvature is also according to constant light speed. This is merely interpreting theory in a consistent manner.

How is it, then, the Schwarzschild metric still differs in form from that of special relativity invariance? This is still just a matter of interpretation, as will be explained.

Apart from polar coordinates the Schwarzschild metric is according to the equation

$$ds^2 = dt^2(1-2@) - \frac{dr^2}{1-2@}$$

In comparison, the special relativity metric with regard to acceleration is of the form

$$ds^2 = dt^2(1-v^2) - dx^2(1-v^2).$$

(The speed of light c is here defined again as unity and does not appear in the equations. The symbol @ denotes the Newtonian gravitational potential usually given as GM/r = v^2, where G is the gravitational constant, M the mass of the gravitational field, and r the distance from the center of the field. The escape velocity of the field is similarly related, being the square root of 2@ = $2v^2$.)

Although these two equations appear different in form, they are convertible by means of a simple operation. The Schwarzschild metric, for instance, converts to the form of special relativity invariance with regard to the insertions of c and c' in the manner

$$ds^2 = dt^2(1-2@c^{-2}) - \frac{dr^2}{c^2(1-2@c^{-2})} = dt^2(1-2@c^{-2}) - \frac{dr^2}{c'^2}(1-2@c^{-2}).$$

The conversion is according to the relation c' = c(1 - 2@)c^{-2}). Note: the fraction v^2/c^2 in the relativistic factor is not affected because gravity affects both light speed and the relative motion of matter proportionally the same.

The ratio c':c implies light speed in a gravitational field is slower by the square of the relativistic factor. The Schwarzschild metric thus reduces to the standard special relativity metric only in the absence of a gravitational field. The two metrics are thus of different form. One is according to constant light speed whereas the other is according to variable light speed. However, there is another way of interpreting the ratio of c':c in a manner consistent with the analogical approach for acceleration. Since speed is nothing more than distance per time, the variance of c' could also be a variance in relative spacetime.

The new interpretation is simply by way of choosing to maintain the relativity of spacetime for invariant light speed for systems in a state of acceleration, either by way of gravity or otherwise. With this choice made as such, the Schwarzschild metric becomes analogous to relative motion with regard to a system in a state of acceleration, not inertial motion, and the condition of invariant light speed in the gravitational field is likewise maintained by defining spacetime parameters of the gravitational field as relatively expanded instead. This relative internal expansion is no different than that of special relativity conditions except it is internal instead of external. Because of slower clocks, the spacetime coordinates of an observer increasing in relative motion are relatively extended.

In effect, because clocks are slower for being in either relative motion or a gravitational field, the retardation of light speed is nullified in both cases.

Spacetime is relatively expanded instead and is of the same form for either relative motion or a gravitational field.

Other Analogies

If special relativity and general relativity are truly of the same form, then all essential relativistic effects of one theory must be analogical to those of the other, not just with regard to spacetime expansion.

The following relativistic effects, in particular, apply with regard to a system in relative motion: Clocks are slower by a relativistic factor. The system is contracted in the direction of relative motion by the same relativistic factor. The mass-energy of the system relatively increases by the same relativistic factor. The spacetime coordinates of the system are relatively expanded by the same relativistic factor. The last effect results merely because a slower clock in relative motion takes more time to mark off coordinates, which are internal events of the system in relative motion as perceived by an observer relatively at rest.

Do the same relativistic effects apply in analogy to those in a gravitational field?

A clock in the field is retarded by the relativistic factor according to the escape velocity of the field. The Schwarzschild metric, however, indicates light speed and relative motion in the field are slower by the square of the relativistic factor, being $1 - 2GM/rc^2$. Nonetheless, by the principle of equivalence, local observers, as falling freely in the field, determine local events, as in a relatively homogeneous field, the same as in gravitational free space. Light speed and relative motion are thus locally the same in gravitational free fall as they are in gravitational free space. In this sense, a consistent interpretation of the two theories is already established.

How do other relativistic effects consistently apply?

Assume the local system is also contracted toward its center by the relativistic factor relative to non-local observers apart from the field. The Earth's moon, for instance, orbits Earth at less distance and slower speed relative to observers outside the solar system. However, to Earthlings whose clocks are relatively slower and whose measuring sticks are relatively shorter, there is no difference to be aware of. The shorter distance and the slower clock combine in effect to nullify the slower speed of the moon by the square of the relativistic factor.

This nullification only applies to the internal comparison of local events. There is, for instance, less relativistic contraction for the distance between the planets Earth and Mars than between Earth and its moon. For a reason to be explained, Mars independently lies more outside the spacetime curvature of the Earth-moon system. It is thus more of a separate system moving independently of Earth. Being their distance of separation is less contracted but the means of measuring distance by Earthlings is more contracted, the distance between Mars and Earth is relatively less contracted than that between Earth and its moon, as in comparison to them being located outside the solar system. The space between Earth and Mars is thus relatively expanded with regard to Earthlings in the field.

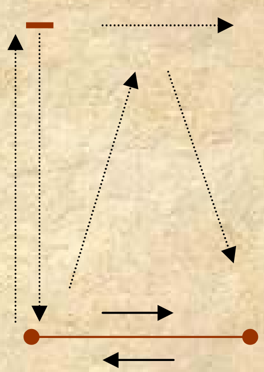
This non-contracting effect of relative spacetime is what actually constitutes the internal relativistic expansion of relative spacetime beyond its relativistic increase in mass. All that is needed is for it to be analogous to the relative spacetime conditions of special relativity.

Is there an analogy of relative expansion of the gravitational field with regard to relative motion? There is indeed with regard to acceleration, which is now analyzed with regard to constant light speed and then with regard to internal spacetime expansion.

The Analogy of Acceleration

Although a system in free fall can be internally compared with an inertial system in relative motion, the Schwarzschild metric actually constitutes a way of comparing systems in and out of the field. The internal condition of gravitational free fall being compared with inertial motion does not apply in this sense. Gravity, in effect, is a state of acceleration and whatever analogy there is with relative motion also pertains to a state of acceleration.

The analogy of acceleration can be identified with regard to the "clock paradox." By the first postulate of special relativity, known as the principle of covariance, there is no distinction to be made between relative motion and rest; for, what is at rests to one observer is relative motion to another. Two observers in relative motion thus perceive the clock of the other as slow. However, if one observer changes direction to return to the other, then it is found from their reunion the clock that changed in direction was slow. This is the "clock paradox." It occurs because of acceleration, as with regard to a change in direction.



How the "clock paradox" applies is illustrated in view of a light signal emitted at the time of departure, say perpendicular to the time of departure, and reflected back to be received at the time of reunion. Since the two observers' clocks are not the same, they disagree on the time light moved during the event. Since distance, as according to theory, is not affected perpendicular to the direction of relative motion, it can be said the slower clock determines a faster light speed. However, there is also another interpretation to consider whereby constant light speed is maintained by simply recognizing the asymmetrical condition of acceleration.

More in general, the measure of relative speeds is relatively affected by acceleration. An even slower clock would have measured a relatively faster light speed had the traveling observer moved faster and gone farther. This is not a contradiction of theory, however, for the departed observer corrects the slower clock to the time of the stay-at-home clock.

Events affected by gravity are similarly in a state of acceleration, the conditions of which apply in comparison with local and non-local systems. Local observers inside a gravitational field, who have slower clocks, perceive non-local events outside the field as occurring relatively faster. In being consistent with constant light speed, they occur relatively faster because they are relatively closer to each other.

This comparison, however, is not new. The real comparison of significance is with regard to local and non-local events within the field where the relatively longer distance of separation between non-local ones, say Mars and Earth, affects the relative magnitude of the gravitational potential of the sun where they reside. For a reason to be explained, the distance between them is relatively longer than if they were the same as an Earth-moon system. Because of this relative increase in distance, the measure of motion is contrary to that of speeds being relatively slower by the square of the relativistic factor and local clocks only being slow by the relativistic factor. Because clocks are only slow by the relativistic factor, instead of the square of the relativistic factor, and because of them moving a shorter distance with relativistic spacetime contraction of the local system, non-local events in the field are perceived as relatively extended.

It is thus the non-local internal conditions of the field that still need to be shown analogical to the relative spacetime conditions of special relativity. There is an analogy to consider, for instance, with regard to the addition of velocities theorem. The internal spacetime expansion of the field allows for an analogical addition of gravitational potentials theorem as well.

Internal Spacetime Expansion

The analogy of spacetime expansion in the field is pertinent with regard to the addition of velocities theorem, as adding them with regard to relative motion is analogous to adding escape velocities of the gravitational field. This analogy is also with regard to spacetime expanding in addition to that of a relativistic increase in mass.

Although the analogy of this aspect of spacetime expansion is with regard to a limiting aspect of the gravitational potential, there is also an analogy of relevance to consider with regard to two independent systems accelerating nearly together at the same rate in the same place and time, one being right behind the other. As they both increase together in speed relative to another observer designated to be relatively at rest, each system relatively contracts in the direction of relative motion. However, their distance of separation does not necessarily contract relative to the other observer remaining relatively at rest. There is no relativistic contraction between them if their relative departures from relatively at rest were relatively simultaneous.

Particularly pertinent to the analysis is how local observers of the systems in acceleration perceive the event. In effect, a local observer of either system perceives no contraction of either system, as the observers in relative motion and their measuring sticks are contracted as well. What is observed instead is an expansion between systems, as also resulting from either the contraction of measuring sticks or the retardation of clocks that determine distance and light speed. In effect, the distance of separation between the two systems in motion does not change relative to a non-local observer but expands relative to the local observers in relative motion.

Systems falling from gravitational free space into a gravitational field are analogous. They become relatively expanded relative to observers already in the field when they enter into it separately from any other system. They are relatively more subject to the gravitational field than they are to each other and therefore constitute separate internal systems of relative motion. When entering separately into a field, as such, they become relatively expanded in the field relative to each other.

Does, then, an earth-moon system expand relative to local observers when it enters into a gravitational field? No. In referring back to the two independent systems accelerating simultaneously, suppose they are linked together by a steel rod. Hence they are no longer independent of each other and their point of departure in time is such that their distance from each other contracts along with the steel rod. The earth-moon system is similar. It is linked together by the earth and moon's mutual gravitational attraction toward each other. They enter into the gravitational field at a common point of entry, but other systems not being gravitationally attached are, as due to other influences, independent in their directions of relative motion. The mere inclusion of their additional mass changes the condition of the field whereas the internal effect of two or more masses falling uniformly does not change. Spacetime thereby expands by the addition of mass entering into the field from a non-common point of entry because the new mass changes the internal condition of the field.

The analogy is consistent with the Schwarzschild metric, but is, in effect, an additional aspect. The Schwarzschild metric specifies the condition of matter free space. In other words, there is no change in the field due to matter entering into it, thus allowing for an interpretation of events in a non-changing field. However, although the metric stipulates the mass in reference is negligible, as not to change the condition of the field, the internal spacetime expansion analogy still applies in the sense it has been predetermined.

Spacetime, as according to the metric, is thus relatively expanded according to the square of the relativistic factor. The internal event of the moon orbiting Earth is also contracted by the relativistic factor. This contraction and the Earthlings' clocks being slow by the relativistic factor, as with regard to the sun's influence, nullify that part of the relativistic spacetime expansion of the solar system. The nullification is in compliance with the principle of equivalence, as with regard to internal conditions of free fall. However, there is still relativistic spacetime expansion between Earth and, say, Mars to consider where only slower clocks affect relativistic spacetime expansion. Because there is no relativistic contraction between Earth and Mars, the internal spacetime expansion between them is only by the relativistic factor, not by the square of the relativistic factor.

This internal spacetime expansion analogy has particular significance with regard to an increase in mass. Instead of only one earth-moon system, consider many of them falling freely toward each other. Each is a system in their own right and all of them together comprise one system decreasing in volume and increasing in density. Now, by the principle of equivalence, there are no internal effects of an earth-moon system in free fall to be aware of locally if the gravitational field is relatively homogeneous. However, the escape velocities of the total field increase along with an increase in mass density and decrease in radius.

Applying an addition of escape velocities theorem in analogy to the addition of velocities theorem indicates a relativistic decrease in the gravitational potential of the system as a whole, as with regard to its influence on itself as a whole. However, there is no change in the individual masses of the system with regard to their gravitational influence on each other and this gravitational influence of internal masses being different than their gravitational influence as a whole constitutes a paradox. However, this paradox is resolved with regard to the internal spacetime expansion of the gravitational field. If spacetime of the field expands in addition to that already specified, then the escape velocities of the field are indeed relatively less. The laws of physics, particularly with regard to gravity, are therefore maintained.

Constant Acceleration

Analogous to the addition of velocities theorem of special relativity is adding escape velocities of a gravitational field. In all practicality, however, a better analogy is more directly associated with constant acceleration (change in speed), as the process of continually adding either velocities or escape velocities is tediously complex. If system P constantly increases relative speed toward that of light, for instance, the relative amount of increase in the same amount of time relatively decreases. This relative decrease in changing speed is directly associated with the time of an observer relatively at rest whereas the addition of velocities theorem only relates velocities according to the spacetime coordinates of two different observers. Although the theorem still applies, in principle, the calculation for all relative speeds in comparison to all other relative speeds requires a different set of values for each change in speed, whereas a more practical formula simply relates all relative speeds to the spacetime coordinates of a particular observer designated as relatively at rest.

A formula for constant acceleration can be derived whereby the addition of velocities theorem is verifiable by example. It is derived for relative spacetime as a modification of the Classical-Newtonian formula for absolute time and absolute space.

The amount of change in velocity from a constant change in speed is determined by a general equation according to Classical-Newtonian Mechanics. It is simply the product of the change in speed and total time of change per interval of time:

$$a(\Delta t) = \Delta v$$

Although this formula pertains to Newtonian Mechanics, it could, as in view of further modification, also be representative of relative spacetime conditions. The magnitude of Δv thus differs according to which theory determines it, as to whether or not it is subject to the addition of velocities theorem.

The possible Newtonian value of the resulting velocity from acceleration being unlimited by the formula above is contrary to the condition of special relativity stipulating no information of events can propagate faster than light speed in a vacuum. The addition of velocities theorem therefore applies. However, saying again, applying it to constant acceleration is tediously complex. The velocity of an object in constant acceleration after two seconds can easily be determined if the velocity is known after one second, but the velocity for any given time entails a lot of calculation. A more convenient form of the theorem is thus derived for constant acceleration in relative spacetime.

It is first convenient to redefine what constitutes constant change in speed in view of relative spacetime. It is here defined in a manner consistent with the principle of covariance. In being consistent as such, system P is determined to be in constant acceleration if the event of accelerating from the velocity of inertial system A to the velocity of inertial system B is the same as accelerating from the velocity of inertial system B to the velocity of inertial system C: providing the velocity of system C relative to an observer B relatively at rest in system B is the same as the velocity of system B relative to an observer A relatively at rest in system A. However, the relative rate of constant acceleration still depends on the difference in velocity of either system B or C relative to respective observers of systems A and B, as it is still conditional to the addition of velocities theorem. The relative rate of constant acceleration thus relatively decreases as relative motion increases.

For the definition to be complete, it needs to specify a condition by which all observers agree on what is constant with regard to how much a system changes relative to a particular inertial system. It could, for instance, simply define the rate of constant acceleration as the change in velocity during one time interval, as determined by an observer relatively at rest in the inertial system from which the accelerating system changed. However, this arbitrary definition is not representative of the maximum, or absolute, rate of acceleration, which is be mathematically determinable.

In relative spacetime, the clock of an observer constantly increasing relative speed becomes progressively slower relative to the clock of the non-accelerating observer. The relative change in velocity is also progressively slower. Conversely the difference in time and change in velocity is more the same when the difference in velocities is more the same. The absolute, or maximum, rate of acceleration thus pertains to the infinitesimal difference in velocities. It still varies, however, with regard to the inertial system with which the clock of the accelerating observer is to be compared. The increment of time of observer A, for instance, relatively differs from the increment of time of observer B. The change in velocity when observer P is momentarily coincident with observer B is thus relatively different than when observer P is momentarily coincident with observer A. These two events are nonetheless comparable in the manner of transformation equations for relative spacetime events.

Let the absolute, or maximum, rate of acceleration relate to the infinitesimal difference in velocities in the manner

$$a = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - t_0} = \frac{v}{\tau}$$

The difference of $v - v_0$ merely becomes the infinitesimal change in velocity occurring in an increment of time τ. The change in velocity becoming v also indicates it changed from being relatively at rest.

The formula above only relates to absolute acceleration in terms of absolute space and absolute time. The product $a\tau$ could therefore determines a range of possible values for v that are unlimited, even of those greater than light speed. In contrast, as for relative spacetime coordinates, the instantaneous rate is relative; it varies along with the relative clock of the observer in relative motion and also the relative rate of acceleration, as a greater rate in change in speed determines a shorter instance of time for the velocity of the accelerating system to be momentarily the same as the inertial system. Moreover, since relative spacetime relatively expands, the relative increase in speed becomes relatively more gradual as it approaches toward the unattainable limit of light speed. The task at hand is thus to replace absolute space and absolute time with relative spacetime parameters.

Simply let $v - v_0 = v$ be the change in velocity of system P changing from being coincident with the relative spacetime of observer A to being coincident with the relative spacetime of observer B. The relative time of this event is to be related to light speed. For this relation, further let x of system A be one unit distance in relation to unit light speed, c = 1, such that the change in velocity occurs in time x/c = x = 1 relative to observer A. This unit distance is arbitrary in the sense it depends on the chosen unit of time (as second, minute, hour or whatever) but could still be the proper increment for absolute acceleration. This arbitrariness allows for an extension of absolute acceleration in constant time. The same time, however, will be shown to be xα relative to observer B such that *a*xα becomes the result of relative acceleration of system P changing from being coincident with observer A to being coincident with observer B.

In this relation, observer B is not initially coincident with either observer A or system P, but moves the distance x in time $t_1 - t_0 = t$ according to observer A. Since v is less than c, the coordinate time t at which observer B moves relative to observer A is thus greater than unity. The event of system P becoming coincident with observer B is nonetheless one subject to Lorentz transformations, two of which, in this case, are

$$x' = \frac{x - vt}{\alpha} = \frac{x - v\,{}^{x}\!/_{v}}{\alpha} = 0$$

$$t = \frac{t' + vx'}{\alpha} = \frac{t'}{\alpha}.$$

By the second transformation, t = t'/α; by the first, x' = 0. That x' is zero is merely consistent with the nature of the event of system P being coincident with observer B at coordinate distance x' = 0 from observer B. Because the time coordinate of observer B is relatively longer, and because the event occurs according to the time coordinate of observer A, the time of this event according to observer B is thus merely shorter by the relativistic factor. Hence t' = tα.

The previous formula for absolute acceleration becomes

$$a = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{\tau\alpha} = \frac{v}{x\alpha} = \frac{{}^{x\alpha}\!/_{t\alpha}}{x\alpha} = \frac{1}{t\alpha}.$$

The product *a*tα is thus unity, as unit velocity. Hence, if *a* = 3/4 distance units per time units per time units, for instance, then tα = 4/3 time units and v = *a*xα = $(3/4)(1)[1 - (3/5)^2]^{1/2}$ = 3/5 distance units per time unit. The latter is equally the same as one distance unit per 5/3 time units.

Relating these values in view of Lorentz transformations further shows

$$x' = \frac{x - vt}{\alpha} = \frac{1 - {}^{3}\!/_{5} \cdot {}^{5}\!/_{3}}{\alpha} = 0$$

$$t' = \frac{t - vx}{\alpha} = \frac{5/3 - 3/5 \cdot 1}{4/5} = \frac{4}{3}$$

$$x = \frac{x' + vt'}{\alpha} = \frac{0 + 3/5 \cdot 4/3}{4/5} = 1$$

$$t = \frac{t' + vx'}{\alpha} = \frac{4/3 + 3/5 \cdot 0}{4/5} = \frac{5}{3}.$$

Relative spacetime parameters of the example are thus x = 1, t = 5/3 and xα = 4/5, *a* = 3/4 and t’ = tα = 4/3.

Substituting t’ for tα in the formula for acceleration further allows for the determination of relations in terms of invariance of the interval:

$$a = \frac{1}{t'} = \frac{1}{\sqrt{t^2 - x^2}}.$$

$$a^2 = \frac{1}{t^2 - x^2} \qquad a^2\left(t^2 - x^2\right) = 1$$

$$a^2t^2 - a^2x^2 = 1 \qquad a^2t^2 = 1 + a^2x^2$$

$$\frac{a^2}{1 + a^2x^2} = \frac{1}{t^2} \qquad \frac{a^2x^2}{1 + a^2x^2} = \frac{x^2}{t^2} = v^2$$

$$a^2x^2 = v^2\left(1 + a^2x^2\right) \qquad a^2x^2 = v^2 + v^2a^2x^2$$

$$v^2 = \frac{a^2x^2}{1 + a^2x^2}$$

$$a^2x^2 - v^2a^2x^2 = v^2 \qquad a^2x^2\left(1 - v^2\right) = v^2$$

$$a^2 = \frac{v^2}{x^2\left(1 - v^2\right)} = \frac{v^2}{x^2\alpha^2}$$

$$a = \frac{v}{x\alpha} \qquad x = \frac{v}{a\alpha} \qquad \tau = \frac{1}{a\alpha}$$

$$a^2\left(t^2 - x^2\right) = \frac{v^2}{x^2\alpha^2}\left[\frac{1}{a^2\alpha^2} - \frac{v^2}{a^2\alpha^2}\right] = \frac{v^2}{a^2x^2\alpha^2}$$

$$= \left[\frac{x^2\alpha^2}{v^2}\right] \cdot \left[\frac{v^2}{x^2\alpha^2}\right] = 1^2 = a^2t'^2 .$$

These equations work for any value *a* and for any value v less than c, where x represents both the time of acceleration of system P relative to system A and the distance observer B moves at velocity v during time t relative to system A.

The time observer A sees the event of system P being coincident with observer B also depends on the distance of acceleration and the time for light to propagate that distance. The duration of such formula for distance is given in the book *Gravity, Quasi Black Holes and Cosmic Relativity*: pages 268-270 in appendix N. The formula for distance of acceleration with respect to the observer relatively at rest is

$$d = \frac{1-\alpha}{a\alpha} = \frac{1-\sqrt{1-\left(\frac{3}{5}\right)^2}}{\left(\frac{3}{4}\right)\sqrt{1-\left(\frac{3}{5}\right)^2}} = \frac{1}{3}.$$

The relative distance of acceleration for one unit of time is thus 1/3 unit distance. The total time for system P to accelerate and be seen by observer A is x + d = 4/3 time units.

With the distance of acceleration given as such, these relations can now be verified according to the addition of velocities theorem and the principle of covariance. The same event from observer B to observer C, for instance, is according to the velocity of observer C relative to observers A and B. In being covariant, it is v = 3/5 unit distance per time units relative to observer B and v_{12} = 15/17 unit distance per time units relative to observer A:

$$v_{12} = \frac{v_1 + v_2}{1 + v_1 v_2} = \frac{\frac{3}{5}+\frac{3}{5}}{1+\left(\frac{3}{5}\right)\left(\frac{3}{5}\right)} = \frac{15}{17}.$$

The time of acceleration from observer A to observer C relative to observer A is according to the equation

$$x_{12} = \frac{v_{12}}{a\sqrt{1-\left(v_{12}\right)^2}} = \frac{\frac{15}{17}}{\frac{3}{4}\sqrt{1-\left(\frac{15}{17}\right)^2}} = \frac{5}{2}.$$

The time of acceleration from observer B to observer C relative to observer A is according to the relation

$$x_{12} - x = \frac{5}{2} - 1 = \frac{3}{2}.$$

Covariance is according to the total time of acceleration from observer B to observer C and for light to propagate the distance of that time of acceleration. It is also according to the slower clock of observer B, the contracted coordinate length in the direction of relative motion, and the relative rate c + v at which light approaches observer B according to observer A:

$$\alpha\left[\left(x_{12}-x\right)+\frac{\alpha d}{c+v}\right] = \frac{4}{5}\left[\left(\frac{5}{2}-1\right)+\frac{\left(\frac{4}{5}\right)\left(\frac{1}{3}\right)}{1+\frac{3}{5}}\right] = \frac{4}{3}.$$

The events are thus covariance in view of the addition of velocities theorem.

The Escape Velocity

The main objective of the analogical approach is with regard to the identity 2GM/r as the escape velocity. This is a Newtonian identity with regard to absolute space and absolute time. It requires relativistic modification with regard to relative spacetime curvature. However, this form of the escape velocity still has local validity for infinitesimal increments in a relatively homogeneous gravitational field. Further connection for the analogical approach requires a consistent interpretation of relativistic kinetic energy.

The Newtonian escape velocity relates to kinetic energy by way of relating the latter, $\frac{1}{2}mv^2$, to the Newtonian gravitational potential, GMm/r, of mass m at distance r from the center of the field of mass M. Hence

$$\tfrac{1}{2}mv^2 = \frac{GMm}{r}$$

$$\frac{2GM}{r} = v^2 .$$

In effect, the product of m and twice the Newtonian potential equals twice the kinetic energy.

To the contrary, the established interpretation of the relativistic kinetic energy is defined as the difference between the total mass-energy of the relative mass m in relative motion and the internal mass-energy of its rest mass:

$$mc^2 - m_o c^2 \approx \tfrac{1}{2} m_o v^2 .$$

This approximation comes by way of the approximation

$$\frac{m_o c^2}{\sqrt{1 - v^2/c^2}} \approx m_o c^2 + m_o c^2 \left[\frac{v^2}{2c^2} \right] = m_o c^2 + \tfrac{1}{2} m_o v^2 .$$

However, it is obvious this manner of defining the relativistic kinetic energy is inconsistent with how the Newtonian kinetic energy relates to the escape velocity.

Although the relativistic increase $mc^2 - m_oc^2$ in internal mass-energy of the rest mass due to relative motion approximates to $\frac{1}{2}m_ov^2$ at speeds considerably less than that of light, the relativistic kinetic energy defined in this way is not the same kinetic energy defined according to Newtonian Mechanics. The Newtonian definition is one-half the product of mass and velocity-squared whereas the relativistic approximation relates to only the increased portion of the mass. However, this increased mass is still the internal energy constitutive of mass. If the total mass were annihilated by converting into light energy, for instance, this light energy of the relative mass is greater than the light energy of the rest mass.

In contrast, the kinetic energy is the external energy of all the mass in relative motion, not just the increased portion of mass. Relativistic kinetic energy is thus more consistently defined in the same way relativistic momentum mv/α modifies Newtonian momentum mv. It is simply a relativistic increase in mass.

This form of kinetic energy is consistent with relativistic invariance:

$$m^2c^2 - m^2v^2 = m\left(mc^2 - mv^2\right).$$

The last quantity in the parenthesis on the right side of the equation, along with the negative sign, is the potential energy in a conservative field of force. Bohr used it as such for deriving quantum restrictions with regard to the electron mass in his atomic model. It is also equal to twice the Newtonian kinetic energy. The form of established relativistic definition of kinetic energy derives from the same invariance, but its identity is still a matter of semantics.

Let the relativistic kinetic energy simply be $\frac{1}{2}m_o v^2/\alpha$. Let the relativistic Newtonian potential of m in the gravitational field of mass M simply be GMm/rα at distance r relative to a local observer of the field. Hence $2GM/r = v^2$. In this way, both the square of the Newtonian escape velocity and the quantity $2GM/rc^2$ in the relativistic factor of the Schwarzschild metric are approximations. The latter is still valid because of approximations canceling each other. If an object moves relatively slower in a gravitational field, for instance, light also moves relatively slower. (Actually, they move a shorter distance relative to expanded spacetime. Since they are in ratio to each other in the relativistic factor, their relativistic modifications cancel.) However, there is a relativistic modification still to consider with the mass M of the field depending on whether it relates to a local observer or one outside the field.

Constant Force

Of particular significance with regard to the various formula for constant acceleration is the square of the velocity obtained from zero to v in time x at the constant rate of acceleration *a* is

$$v^2 = \frac{a^2 x^2}{1 + a^2 \frac{x^2}{c^2}}$$

The velocity v is according to relative spacetime. In comparison, the velocity for absolute space and absolute time of Newtonian Mechanics is

$$ax = \frac{v}{\sqrt{1 - v^2 c^{-2}}}\,.$$

The velocity v is thus obtained from constant acceleration according to relative spacetime where x is the relative time of a system relatively at rest to which any value of v is compared.

This comparison is not merely with regard to acceleration; it is also with regard to force. Constant force determines constant acceleration: such that

$$Ft = mat = m'v = mv\alpha^{-1} \qquad \alpha = \sqrt{1 - v^2 c^{-2}} \qquad x = t \qquad m' = m\alpha^{-1}\,.$$

These equations show the change in momentum is the same for either relative spacetime or absolute space and absolute time where the change in velocity being less for relative spacetime is compensated for by the relative mass being relatively greater.

These relations here given are with regard to the gravitational potential being analogous to relative motion. However, relative motion in a gravitational field is slow by the square of the relativistic factor, not by just the relativistic factor. Nonetheless, the product of mass and velocity squared further associates with the kinetic energy rather than momentum. It is thus assumed the *kinetic potential* associates with relative mass and spacetime expansion of the gravitational field. It is here defined as

$$\tfrac{1}{2}m(at)^2 .$$

The *kinetic potential* is thus merely the classical kinetic energy that would occur if mass did not relatively increase with relative motion.

For the gravitational analogy of the *kinetic potential*, let v√2 be the escape velocity in relation to the rate of rotational-gravitational free fall such that $2v^2$ is twice the Newtonian gravitational potential @ with regard to spacetime curvature. The product *a*t is comparable to an escape velocity with regard to absolute space and absolute time such that the equivalent of a^2t^2 is $2@_+ = 2GM/r'$, where r' is greater than r by the relativistic factor. Substituting these values in the previous equation obtains the relation

$$2@_+ = \frac{2@}{1-2@} .$$

Further substituting GM/r for @ and dividing by 2 obtains

$$@_+ = \frac{GM}{r(1-2@)} .$$

This equation in analogy to a mass in constant acceleration implies a relativistic increase in relative mass compensates for a relativistic decrease in gravitational potential. This result is only feasible if the relativistic increase in internal spacetime of the gravitational field is greater than the relativistic increase in the mass of the field.

An increase in relative mass of the field and relative decrease in gravitational potential is analogous to the increase in relative mass and relative decrease in *kinetic potential* of a mass constantly increasing in relative speed. It is also consistent with the condition of relative spacetime expansion of the field along with an increase in mass relative to non-local observers apart from the field. Relative to them, spacetime of the field expands in contrast to light speed being decreased by the square of the relativistic factor. An increase in relative mass is thereby required in order to maintain the same ratio of mass to distance according to the ratio GM/R being equivalent to the square of the orbital velocity v.

The final analogy is thus with regard to relative mass. Special relativity requires a mass in relative motion to be relatively increased along with a relative increase in spacetime. The established interpretation of general relativity also requires the same for a mass in a gravitational field. This analogous condition along with the analogous conditions of spacetime expansion complies as well, thus maintaining consistency of theory.

Comparing Metrics

By substituting c' for c, the form of the Schwarzschild metric becomes

$$ds^2 = dt^2(1-2@) - dr^2(1-2@).$$

It is thus comparable to a special relativity metric for acceleration of the form

$$ds^2 = dt^2(1-v^2) - dx^2(1-v^2).$$

But the metric used for the derivation of equations for constant acceleration is of standard special relativity invariance:

$$s^2 = t^2 - x^2 = t'^2 - x'^2.$$

The deduction of relative spacetime of general relativity to that of the relative spacetime of special relativity is therefore not complete.

The equations derived for constant acceleration are with regard to spacetime coordinates of an observer relatively at rest whereas the Schwarzschild metric compares the coordinates of an observer in a gravitational field to those of an observer in gravitational free space. A true comparison is thus with regard to the spacetime coordinates of a system in constant acceleration to one relatively at rest. An analogical comparison has been made with regard to the clock paradox, as both the returning observer and one statically at rest in a gravitational field perceive the clock of the stay at home observer relatively at rest in gravitational free space as fast. A circling observer perceives likewise. There is an analogy also with regard to constant acceleration.

The analogy relates mathematically if the form of the metric for constant acceleration being the same as the deduced form of the Schwarzschild metric. This sameness in form is partially achieved by substituting the parameters t'_2, t_2 and x_2 in place of the previous parameters t', t and x, respectively. Standard invariance is thus replaced with the relation

$$a = \frac{1}{t'_2\sqrt{1-v^2}} = \frac{1}{\sqrt{t_2^2(1-v^2) - x_2^2(1-v^2)}}.$$

By multiplying all equalities by $\alpha = \sqrt{(1-v^2)}$, the equation becomes

$$a\alpha = \frac{1}{t'_2} = \frac{1}{\sqrt{t_2^2 - x_2^2}}.$$

A previous equations derived from standard invariance is

$$ax_2 = \frac{v_{12}}{\alpha_{12}} = \frac{2v}{(1+v^2)\sqrt{1-\frac{(2v)^2}{(1+v^2)^2}}} = \frac{2v}{\sqrt{(1+v^2)^2 - 4v^2}}$$

$$=\frac{2v}{\sqrt{1+2v^2+v^4-4v^2}}=\frac{2v}{\sqrt{1-2v^2+v^4}}=\frac{2v}{\sqrt{\left(1-v^2\right)^2}}=\frac{2v}{1-v^2}=\frac{2v}{\alpha^2}.$$

The equality on the far right is the velocity obtained for absolute space and absolute time whereas the relative velocity is indicated as $ax_2\alpha^2 = 2v$.

The analogy is still not complete, as there remain discrepancies with regard to time, velocity and light speed. The duration x_2/c, for instance, relates to $2x/c$. The latter is two units distance per light speed instead of one unit distance per light speed. The Schwarzschild metric generally compares events in a gravitational field with those relatively at rest in gravitational free space and the comparison is with regard to one-unit time intervals, whether they are in relation to seconds, minutes or whatever. The metric for constant acceleration should somehow be comparable to a single time interval in relation to v instead of 2v. It should also indicate variable light speed in the way c' has been substituted for c in the Schwarzschild metric. However, such conditions are indeed explainable in view of the mathematical results.

For a conceptual interpretation of this mathematical relation, consider the event of system P changing from velocity –v to velocity 0 in time x/c relative to observer A in a way that observer P changes from being coincident with observer B_1 to becoming coincident with observer A. Observer P continues to accelerate at the same rate to become coincident with observer B_2 after changing from velocity 0 to velocity v relative to observer A. The total change in velocity is thus 2v during time 2x relative to observer A.

Observer P continues to accelerate at the same rate to become coincident with observer C. Relative to observer A, the total change from being coincident with observer A to becoming coincident with observer C is from velocity 0 to velocity v_{12} during time x_2. However, in compliance with the principle of covariance, the event relative to observer B_2 is the same as for observer P changing from being coincident with observer B_1 to becoming coincident with observer B_2. The total change in velocity is thus 2v during time 2x relative to the slower clock of observer B_2.

The clock of observer P is relatively the same as the clock of either observer B_1, A, B_2 or C whenever the instantaneous velocity of observer P is momentarily the same, respectively, as the velocity of either observer B_1, A, B_2 or C. The total time of observer P's clock during the event, however, is not determined by these equations. However, the total time of observer P's clock is not the issue at stake; the metrics only compare the increments of time at the respective moments. The increment of time when observer P is coincident with observer B is thus the same for observer P as it is for observer B_2. Invariance of the interval in relation to the parameters t'_2, t_2 and x_2 is therefore with regard to the increment of time of observer P in comparison with the increment of time of observer A for the same event.

The explanation is with regard to relating the time t_2 and total change in velocity $\Delta v = v - (-v) = 2v$ to two equal events: similar to the clock paradox whereby the traveling observer is there-and-back. Observer A is representative of the stay-at-home observer and observers B_1 and B_2 are representative of the to-there-and-back condition of the traveling observer. Being the duration to there is the same as the duration from back to there, the time and velocity from either to there or back equates with the time t and velocity v used for standard invariance.

The condition the invariance relation for constant acceleration is valid for the parameters t and v as well as for the parameters t_2 and 2v is fairly obvious with regard to the acceleration of observer P from observer B_1 to observer B_2 relative to observer A. The principle of covariance further requires it to be true with regard to observer P accelerating from observer A to observer C relative to observer B_2. In effect, equations for constant acceleration are derivable from both standard and relativistic modified invariance. The only essential difference between the two methods is the latter is applicable to the viewpoint of the accelerated observer whereas standard invariance alone only applies to observers perceiving themselves relatively at rest.

The equation $c' = c$ is used only as a conversion factor. Although using one equality or the other changes the form of the equation, the total value does not change. The conversion factor is applicable to the acceleration metric in the sense the to-there-and-back observer having a slower clock thus travels a relatively longer distance but still measures the duration of events to be relatively shorter. The observations are thus analogically the same with regard to the increments dt and dr in the Schwarzschild metric being, respectively, relatively shorter and relatively expanded by the relativistic factor.

Cosmic Implications

Consider a model of the universe proposed by Einstein that constitutes a finite universe curving in on itself in such a way its center is determined relative to the position of the observer. Also consider the observer could just as well be at the edge of the universe where clocks are relatively retarded. This possibility has cosmic implications with regard to the singularity and the black hole.

Generally, as apart from closed spacetime curvature, a gravitational field increases toward the surface of mass and decreases either away from the mass or inside the spherical volume of mass. The gravitational forces from various directions cancel in total gravitational effect inside the sphere. They cancel completely at the center such that there is zero gravitational potential. The gravitational potential of a mass is thus greater at its outer edge than towards its center. This condition applies to the universe as a whole providing the universe is relatively finite. Clocks at the center are therefore not relatively affected.

Or are they?

Who is to say we are either at the center of the universe or at its outer edge?

Einstein's model of the universe is of closed curvature. The relativistic effects of the gravitational potential are smoothed out to apply everywhere in it. However, it would also seem the gravitational potential in a universe of closed curvature is great enough for the relativistic effect on clocks to be completely stopped. This assumption cannot be true, as everything coming to a halt is contrary to a sacred law of physics—conservation of energy—with which special relativity and general relativity should conform. The relativistic effect of clock retardation must somehow be nullified for the observer to be at the center of the universe.

A nullification of clock retardation is not only possible; it is reasonably consistent with Einstein's cosmology. As gravitational effect is canceled at the center of mass, and as spacetime curves in on itself from a sufficient amount of finite mass, gravity negates itself. Such relativistic effect as clock retardation is therefore canceled. However, this negation also implies spacetime curvature, in general, has a relativistic reduction in effect in order for clocks to become more retarded by a greater mass and eventually become less retarded by an even greater mass.

If the relativistic increase and relativistic decrease are of the same opposite order, which is a reasonable assumption, then the increase and decrease of relativistic effect cancel out somewhere between the two extremes. A unique value is an escape velocity of the square root of one-half light speed. Let $2GM/r = \frac{1}{2}c^2$. One-half of $2GM/r$ is $GM/r = \frac{1}{4}c^2$, being the square of the orbital speed at the critical distance where the escape velocity is $c\sqrt{\frac{1}{2}}$. The orbital speed is thus the square root of $\frac{1}{4}c^2$, which is $\frac{1}{2}c$, but this value is subject to relativistic expansion. It is thus increased by the relativistic factor.

The relativistic factor is the square root of $1 - 2GM/rc^2$. Somehow it has been prevented from becoming zero, whereby $1 - 2GM/rc^2 \neq 1 - c^2/c^2 = 1 - 1 = 0$. By reason just given, it equals the square root of $1 - \frac{1}{2} = \frac{1}{2}$. Since the orbital speed of ½ light speed is subject to the relativistic factor for expansion, dividing ½ by the square root of ½ gives the square root of ½: for

$$\frac{\frac{1}{2}}{\sqrt{\frac{1}{2}}} = \sqrt{\frac{\left(\frac{1}{2}\right)^2}{\left(\sqrt{\frac{1}{2}}\right)^2}} = \sqrt{\frac{\frac{1}{4}}{\frac{1}{2}}} = \sqrt{\frac{1}{4} \times \frac{2}{1}} = \sqrt{\frac{1}{2}}\,.$$

The orbital speed relative to a local observer is therefore the same as the escape velocity relative to a non-local observer situated outside the field. Moreover, relative to a local observer inside the field, whose relative spacetime is relatively expanded, the escape velocity is the square root of one half divided by the square root of one half, which is unity, which is light speed. The critical Schwarzschild radius at the limit c is therefore subject to relative internal spacetime expansion.

The square root of ½ is unique inasmuch as it determines the escape velocity in relation to the orbital speed of the same distance. It is thus questionable as to whether the critical value c pertains to the orbital speed or the escape velocity at the edge of the universe. In either case, observers situated anywhere within the relative spacetime continuum perceive the distance as the limiting condition of an escape velocity at light speed.

In this interpretation of Einstein's cosmology there is no possibility of a singularity, but the black hole is still possible. The singularity does not exist because the internal perception of the limit c is only ½c; wherefore clocks do not slow to a halt. However, the black hole is still evident insofar as the Schwarzschild radius represents an attainable limit where a visible finite part of an infinite universe can be closed. In this sense, there can be an unlimited number of universes within universes. This result is merely achieved if the spacetimes of special and general relativity are analogically of the same form.